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Thermodynamics of $f(R)$ Gravity with Disformal Transformation

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Abstract

We study thermodynamics in $f(R)$ gravity with the disformal transformation. The transformation applied to the matter Lagrangian has the form of $\gamma_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi$ with the assumption of the Minkowski matter metric $\gamma_{\mu\nu} = \eta_{\mu\nu}$, where ϕ is the disformal scalar and X is the corresponding kinetic term of ϕ . We verify the generalized first and second laws of thermodynamics in this disformal type of $f(R)$ gravity in the Friedmann-Lemaître-Robertson-Walker (FLRW) universe. In addition, we show that the Hubble parameter contains the disformal-induced terms, which define the effectively varying equations of state for matter.

Introduction

- In 1995, T. Jacobson further showed that Einstein equation can be derived from the thermodynamic behavior of spacetime. [1]
- In 2005, Cai and Kim demonstrated that the Friedmann equations can be derived from the first law of thermodynamics on the apparent horizon of the universe.[2]
- Connection between thermodynamics and $f(R)$ gravity and other modified gravity has been widely investigated. [3-5]
- Bekenstein argued that two metrics, the physical metric and the gravitational metric, should be related through the **disformal transformation** in order to satisfy equivalence principle and causality: [6]

$$\gamma_{\mu\nu} = A(\phi, X)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi. \quad (1)$$

- We examine thermodynamic properties in $f(R)$ gravity including the disformal transformation with the following assumptions.

Assumption

1. Consider the flat FLRW universe ($k=0$).
2. Consider the physical metric to be Minkowskian: $\gamma_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$
3. Consider the physical metric directly coupled to matter only.
4. Interpret the effects of $f(R)$ deviated from GR as dark energy.
5. Assume all the matter including dark energy are perfect fluids.

Action

- The action of $f(R)$ gravity with matter is given by

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + \sum_i S_M^{(i)}[\gamma_{\mu\nu}], \quad (2)$$

- Since the disformal metric couples to the matter action, we have induced some new matter contents (induced energy density ($\rho_i^{(\text{in})}$) and pressure ($P_i^{(\text{in})}$))

$$\rho_i^{(\text{in})} = [(1 - a^2)(3\dot{a}\dot{\phi} + a^3\ddot{\phi})/(3\dot{a}\dot{\phi})]\rho_i \equiv \lambda\rho_i, \\ P_i^{(\text{in})} = -\frac{a\ddot{\phi}(1 - a^2)}{3\dot{a}\dot{\phi}}\rho_i.$$

Thermodynamics in Jordan Frame (non-equilibrium picture)

- Modified Friedmann equations:

$$H^2 = \frac{8\pi G}{3F}(\rho_M + \hat{\rho}_d), \\ \dot{H} = -\frac{4\pi G}{F}(\rho_M + \hat{\rho}_d + P_M + \hat{P}_d). \quad (3)$$

where

$$\hat{\rho}_d = \frac{1}{8\pi G} \left(\frac{1}{2}(FR - f) - 3H\dot{F} \right), \\ \hat{P}_d = \frac{1}{8\pi G} \left(\ddot{F} + 2H\dot{F} - \frac{1}{2}(FR - f) \right), \quad (4)$$

- Continuity equation (energy-momentum non-conservation)

$$\dot{\rho}_{total} + 3H(\hat{\rho}_{total} + \hat{P}_{total}) = \frac{3H^2\dot{F}}{8\pi G} \quad (5)$$

where dot represents derivatives w.r.t. cosmic time t .

- Unified First Law:

$$Td\hat{S} + Td_i\hat{S} = -d\hat{E} + \hat{W}dV, \quad (6)$$

where

T is the temperature on the horizon,

$\hat{S}$ is the entropy on the horizon,

$d_i\hat{S}$ is the entropy change rate of ordinary matter and induced matter within the horizon,

$\hat{E}$ is the total energy within the horizon,

$\hat{W}$ is the work density,

V is the total volume.

- Generalized Second Law:

$$\frac{d\hat{S}}{dt} + \frac{d_i\hat{S}}{dt} + \frac{d\hat{S}_t}{dt} = \frac{12\pi F\dot{H}^2}{GRH^3} \geq 0. \quad (7)$$

provided $F = \partial f(R)/\partial R > 0$,

where $d\hat{S}_t/dt$: entropy change rate of ordinary matter and induced matter within the horizon in non-equilibrium description.

Thermodynamics in Jordan Frame (equilibrium picture)

- Modified Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\bar{\rho}_M + \rho_d + \lambda\bar{\rho}_M), \\ \dot{H} = -4\pi G\left((\bar{\rho}_M + \rho_d + P_r + P_d) + \lambda(1 + w^{(\text{in})})\bar{\rho}_M\right), \quad (8)$$

- Dark energy density and pressure are defined to be:

$$\rho_d = \frac{1}{8\pi G} \left[\frac{1}{2}(FR - f) - 3H\dot{F} + 3H^2(1 - F) \right], \\ P_d = \frac{1}{8\pi G} \left[\ddot{F} + 2H\dot{F} - \frac{1}{2}(FR - f) - (1 - F)(2\dot{H} + 3H^2) \right]. \quad (9)$$

- Energy-momentum conservation:

$$\dot{\rho}_{total} + 3H(\rho_{total} + P_{total}) = 0, \quad (10)$$

- Unified First Law & Generalized Second Law

$$TdS = -dE + WdV \quad (11)$$

$$\frac{d}{dt}(S + S_t) = \frac{12\pi\dot{H}^2}{GRH^3} \geq 0, \quad (12)$$

where $S = A/4G$ is the entropy in equilibrium description and dS_t/dt is entropy change rate of ordinary matter and induced matter within the horizon in equilibrium description.

Thermodynamics in Einstein Frame

- After applying conformal transformation, $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} := \exp(\sqrt{4\pi G/3}\omega)g_{\mu\nu}$, modified Friedmann equations become

$$\tilde{H}^2 = \frac{8\pi G}{3}((\tilde{\rho}_M + \tilde{\rho}_\omega)), \\ \tilde{H}' = -4\pi G((\tilde{\rho}_M + \tilde{\rho}_\omega + \tilde{P}_M + \tilde{P}_\omega)), \quad (13)$$

where “tildes” represent quantities in Einstein frame and

“ $\prime$ ” represent derivatives w.r.t. $\tilde{t}$ with $d\tilde{t} = \Omega dt$ and

$\tilde{\rho}_M = (1 + \lambda)(\bar{\rho}_M + \bar{P}_r)$, $\tilde{\rho}_\omega = (1/2)\omega'^2 + V(\omega)$, $\tilde{P}_\omega = (1/2)\omega'^2 - V(\omega)$.

- Continuity equation

$$\tilde{\rho}'_t + 3\tilde{H}(\tilde{\rho}_t + \tilde{P}_t) = 0,$$

- Unified First Law & Generalized Second Law

$$\tilde{T}d\tilde{S} = -d\tilde{E} + \tilde{W}d\tilde{V} \quad (14)$$

$$\frac{d}{d\tilde{t}}(\tilde{S} + \tilde{S}_t) = \frac{12\pi\tilde{H}'^2}{G\tilde{R}\tilde{H}^3} \geq 0, \quad (15)$$

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